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A super-Ohmic energy absorption in driven quantum chaotic systems

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Abstract. We consider energy absorption by driven chaotic systems of the symplectic symmetry class. According to our analytical perturbative calculation, at the initial stage of evolution the energy growth with time can be faster than linear. This appears to be an analog of weak anti-localization in disordered systems with spin-orbit interaction. Our analytical result is also confirmed by numerical calculations for the symplectic quantum kicked rotor.

PACS. 05.45.Mt Quantum chaos; semiclassical methods – 73.21.La Quantum dots – electron states – 73.23.-b Electronic transport in mesoscopic systems – 73.20.Fz Weak or Anderson localization

1 Introduction

The problem of energy absorption in a system driven by an external time-dependent field is fundamental and important in many areas of modern physics. For a metallic sample of the volume $\mathcal{V}$ in an external electric field $E(t) = E_0 \cos \omega t$ the textbook solution to this problem is given by the expression for the Joule heating in the Ohmic regime: $W_0 = \mathcal{V}\,\sigma_0\, E_0^2/2$, is the *constant* energy absorption rate determined by the Drude conductivity σ_0.

This classical picture is based on the linear response theory for systems with essentially continuous spectrum of electron states. For quantum systems with few degrees of freedom or for mesoscopic systems with many degrees of freedom but still appreciable level separation the Ohmic regime may break down leading to a *time-dependent* absorption rate $W(t)$.

This point can be illustrated by an example of the quantum kicked rotor (QKR) with the Hamiltonian:

$$\hat{H} = \frac{\hat{\ell}^2}{2I} + K\,V(\theta) \sum_{n=-\infty}^{\infty} \delta(t - nT), \quad V(\theta) = \cos\theta, \quad (1)$$

where $\hat{\ell} = -i\partial/\partial\theta$ is the angular momentum, I is the moment of inertia, and K is a constant controlling the strength of perturbation. For generic sufficiently large K the classical dynamics described by the Hamiltonian (1) is completely chaotic. The period-averaged energy absorption rate $W_0 = K^2/(4TI)$ in this case is independent of time, analogously to the Ohmic absorption. Yet at sufficiently long times $t \gg t_* \sim K^2 I^2/T$ the Ohmic regime breaks down because of the accumulation of quantum corrections and the absorption rate decreases to zero. This effect is known as *dynamic localization* (DL) in the energy space [1], and is analogous to Anderson localization for disordered systems [2]. Such behavior is not specific to QKR, it occurs in other chaotic systems [3,4].

However, if $T/(4\pi I)$ takes a rational value, the separation between certain energy levels of the rotor becomes an integer multiple of the frequency $2\pi/T$, and the absorption rate is linear in time: $W(t) \propto t$ [1]. The same takes place for a harmonic oscillator coupled to the external harmonic field via the coordinate, when the frequency of the field is exactly at resonance with the oscillator frequency [5]. Fermi accelerator is another example of a system where $W(t)$ can grow with time [6]. Such an anomalous (*growing* with time) *super-Ohmic* behavior is typical of resonances. In contrast to the localization, it is analogous to the *ballistic* transport through resonant levels in a tight-binding model of 1d crystals. One can trace it back to the classical integrability of the system with a time-dependent perturbation.

In this letter we consider a class of *chaotic* systems without resonances which show the super-Ohmic energy absorption. Namely, we focus on the *quantum corrections* $\delta W(t)$ to the energy absorption rate in the time-dependent random matrix theory (RMT) of the *symplectic* symmetry class, described by the Hamiltonian:

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t), \quad (2)$$

which possesses the time-reversal symmetry, but not the spin-rotation symmetry. Here $\hat{H}_0$ and $\hat{V}$ are random matrices [7] whose symmetry will be specified below,

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and $\phi(t)$ is a given function of time. This model describes e.g. the dynamics of electrons in driven quantum dots in the presence of a spin-orbit interaction.

The corresponding problem for the orthogonal and unitary symmetry classes has been recently considered [8] and *analytical* expressions for $\delta W(t)$ have been obtained. For the harmonic perturbation $\phi(t) = \cos\omega t$ switched on at $t = 0$, the absorption rate $W(t)$ appears to be related to the frequency-dependent diffusion coefficient $D(\omega)$ in a quasi-1d disordered wire of the corresponding symmetry class, with the quantum corrections included:

$$\frac{W(t)}{W_0} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{(-i\omega + 0)} \frac{D(\omega)}{D_0}, \qquad (3)$$

where D_0 is the classical diffusion coefficient. This relationship does not contain any specific feature of the model and is also valid for the QKR in the region of parameters where it can be mapped onto the quasi-1d nonlinear σ-model [9]. If equation (3) is valid in the symplectic case as well, the energy absorption rate $W(t)$ should *grow* with time beyond the Ohmic limit W_0, as $D(\omega)$ is known to have *positive* quantum corrections in the presence of a spin-orbit interaction [10]. Our calculations presented below show that this is indeed the case.

2 Choice of the model

We adopt the following single-electron Hamiltonians, which turns out to be the most convenient technically for perturbative calculations:

$$\hat{H}(t) = \frac{\hat{\mathbf{p}}^2}{2m} + U(\mathbf{r}) + \hat{U}_{so}(\mathbf{r}) + V(\mathbf{r})\,\phi(t), \qquad (4)$$

where $\hat{\mathbf{p}} = -i\nabla$, and $U(\mathbf{r})$ and $V(\mathbf{r})$ are independent Gaussian random fields: $\langle U(\mathbf{r})\,U(\mathbf{r}')\rangle = a_U\delta(\mathbf{r}-\mathbf{r}')$, $\langle V(\mathbf{r})\,V(\mathbf{r}')\rangle = a_V\delta(\mathbf{r}-\mathbf{r}')$. The spin-orbit interaction is also taken to be random [10,11]:

$$\hat{U}_{so}(\mathbf{r}) = \sigma\,[\nabla U_{so}(\mathbf{r}) \times \hat{\mathbf{p}}]\,, \qquad (5)$$

where $\langle U_{so}(\mathbf{r})\,U_{so}(\mathbf{r}')\rangle = a_{so}\delta(\mathbf{r}-\mathbf{r})$, and $\sigma = (\sigma^x, \sigma^y, \sigma^z)$ are Pauli matrices.

The advantage of the model (4) is that the spin-orbit coupling and the driving perturbation are represented by random locally correlated fields. This makes it possible, after a proper re-formulation [12,13] within the Keldysh formalism [14], to apply basic rules of the impurity diagrammatic technique [15] and its extension used in the theory of weak Anderson localization and mesoscopic phenomena [16] to consider essentially nonlinear in the driving perturbation, non-equilibrium problems.

In the absence of the time-dependent term equation (4) is a basic model for describing disordered metals with a random spin-orbit interaction. The kinetic energy term determines the bulk density of states ν (per unit volume, per spin projection). Then $a_U = 1/(2\pi\nu\tau_0)$ and $\overline{[\mathbf{p}\times\mathbf{p}']^2}\,a_{so} = 1/(2\pi\nu\tau_{so})$, where τ_0 is the momentum relaxation time, $\tau_{so} \gg \tau_0$ is the spin relaxation time, and $\overline{[\mathbf{p}\times\mathbf{p}']^2}$ denotes the momentum product averaged over the Fermi surface. It has been shown [17] that for a finite sample in the long-time, low-energy limit $\epsilon \ll 1/\tau_{so} \ll E_{Th}$ (E_{Th} being the Thouless energy) this model reduces to the zero-dimensional nonlinear σ-model which, in turn, is equivalent to the RMT of the symplectic symmetry class. This corresponds to $\hat{H}_0$ in equation (2) being a random matrix from the Gaussian symplectic ensemble (GSE) with the mean energy level separation $\delta = 1/(2\nu\mathcal{V})$.

The equivalence of the model (4) to the time-dependent RMT can be also demonstrated, but it requires additional conditions:

$$\frac{a_V}{a_U} = 4\Gamma\tau_0 \ll \left(\frac{\omega}{E_{Th}}\right)^2 \ll 1. \qquad (6)$$

Here we have introduced a parameter $\Gamma = \pi\nu a_V/2$. We will always be interested in the limit $\delta \ll \Gamma \ll \omega$, in which case $1/\Gamma$ is the time required to absorb one photon, as given by the Fermi Golden Rule. The first condition (6) allows to neglect the multi-photon absorption processes which can easily violate the condition of small energy transfer $\Delta\epsilon \ll E_{Th}$ even when the condition $\omega \ll E_{Th}$ is fulfilled. Under the conditions (6) the last term of equation (4) corresponds to $\hat{V}\phi(t)$ of equation (2) with $\hat{V}$ being a random matrix from the Gaussian *orthogonal* ensemble (GOE), whose matrix element mean square is given by $\Gamma\delta/\pi$.

We note that if instead of the time-dependent perturbation with *random* $V(\mathbf{r})$ one considers a more physical form of the perturbation with some *fixed* $V(\mathbf{r})$ corresponding to a uniform electric field or a modulation of the quantum dot confinement potential, one can show [12,13] that under the conditions (6), where only the basic symmetries (time-reversal, spin-rotation) matter, it will be equivalent to the same RMT with a properly redefined Γ. However, calculations would be quite cumbersome in this case because of the necessity to take into account boundary conditions. The coupling by a random field $V(\mathbf{r})$ with local correlations helps to avoid this technical problem.

We also note that instead of equation (4) one can consider a model where spin-orbit interaction is in the time-dependent term: $\hat{H}(t) = \hat{\mathbf{p}}^2/2m + U(\mathbf{r}) + \hat{U}_{so}(\mathbf{r})\,\phi(t)$. It corresponds to a different kind of the time-dependent RMT, where in equation (2) $\hat{H}_0$ is taken from GOE and $\hat{V}$ from GSE. Still, the result turns out to be the same as for the model (4), being remarkably robust and independent of whether the spin-rotational invariance is broken in the time-independent or in the time-dependent term [see also the discussion after Eq. (12)].

3 Weak dynamic antilocalization

The model (4) allows for a perturbative treatment which is very similar to the theory of weak Anderson localization [16]. The building blocks of this theory are ladder diagrams: the diffuson and the cooperon (Fig. 1). The latter

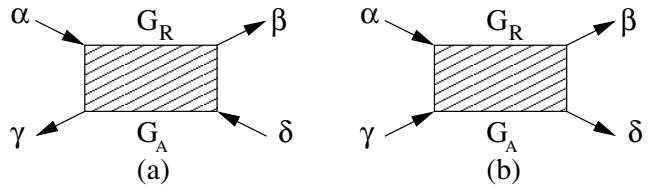

Fig. 1. Diagrammatic representations for (a) diffuson, (b) cooperon; the Greek indices label the spin projections $\uparrow, \downarrow$.

can also be represented as the maximally crossed ("fan") series of diagrams. In general they are functions of time and momentum. However, in the RMT, or ergodic, limit the main contribution to observables is done by the *zero momentum mode*. In this limit the diffuson $\mathcal{D}^{\alpha\beta}_{\gamma\delta}(t,t';\eta)$ and the cooperon $\mathcal{C}^{\alpha\beta}_{\gamma\delta}(\eta,\eta';t)$ are given by:

$$\mathcal{D}^{\alpha\beta}_{\gamma\delta}(t,t';\eta) = (1/2)\, D(t,t';\eta) \times \left[\delta_{\alpha\gamma}\delta_{\delta\beta} + e^{-\frac{4(t-t')}{3\tau_{so}}}\sigma_{\alpha\gamma}\sigma_{\delta\beta}\right], \quad (7)$$

$$\mathcal{C}^{\alpha\beta}_{\gamma\delta}(\eta,\eta';t) = C(\eta,\eta';t) \times \left[\langle\alpha\gamma|\hat{P}_0|\beta\delta\rangle + e^{-\frac{2(\eta-\eta')}{3\tau_{so}}}\langle\alpha\gamma|\hat{P}_1|\beta\delta\rangle\right], \quad (8)$$

where $\hat{P}_{0,1}$ are the projectors on the subspaces with the total spin $S = 0, 1$, respectively:

$$\langle\alpha\gamma|\hat{P}_0|\beta\delta\rangle = \sigma^y_{\alpha\gamma}\sigma^y_{\delta\beta}/2, \quad (9)$$

$$\langle\alpha\gamma|\hat{P}_1|\beta\delta\rangle = \left(\delta_{\alpha\gamma}\delta_{\delta\beta} + \sigma^x_{\alpha\gamma}\sigma^x_{\delta\beta} + \sigma^z_{\alpha\gamma}\sigma^z_{\delta\beta}\right)/2. \quad (10)$$

$D(t,t';\eta)$ and $C(\eta,\eta';t)$ are defined as follows:

$$D(t,t';\eta) = \theta(t-t')\exp\left\{-\int_{t'}^{t}\gamma(t'',\eta)dt''\right\}, \quad (11)$$

$$C(\eta,\eta';t) = \theta(\eta-\eta')\exp\left\{-\frac{1}{2}\int_{\eta'}^{\eta}\gamma(t,\eta'')d\eta''\right\}, \quad (12)$$

where $\gamma(t,\eta)$ is determined by the external field:

$$\gamma(t,\eta) \equiv \Gamma[\phi(t+\eta/2) - \phi(t-\eta/2)]^2. \quad (13)$$

Note that equations (7, 8) retain crossover triplet terms. At times larger than τ_{so} they decay exponentially, and the results reduce to those for the RMT model (2) with $\hat{H}_0$ from GSE, which corresponds to $\tau_{so} \to 0$, so that in this model the triplet term would be absent from the very beginning. The RMT model (2) with $\hat{H}_0$ from GOE and $\hat{V}$ from GSE would correspond to a finite τ_{so}.

The diagrammatic technique in the Keldysh representation allows to calculate the time- and energy-dependent electron distribution function $f(\epsilon,t)$, from which one deduces the energy absorption rate:

$$W(t) = \frac{\partial}{\partial t}\int \epsilon\, f(\epsilon)\, d\epsilon. \quad (14)$$

The expansion in the number of the diffuson or cooperon loops corresponds to the expansion in the powers of the mean level spacing δ, which is assumed to be the smallest energy scale of our problem [8,13]. The leading contribution is given by diagrams containing no diffuson or cooperon loops; it corresponds to the Ohmic absorption with the rate

$$W_0 = \frac{2\Gamma}{\delta}\,\overline{(\partial_t\phi)^2}, \quad (15)$$

where the overline denotes the average over the period. The next order correction to the Ohmic absorption rate is obtained by taking into account one-loop diagrams:

$$W(t) = W_0 + \frac{\Gamma}{\pi}\int_0^t \left(3e^{\frac{-4\eta}{3\tau_{so}}} - 1\right) \times \partial_t\phi(t)\,\partial_t\phi(t-\eta)\,C(\eta,-\eta;t-\eta/2)\,d\eta. \quad (16)$$

For $\phi(t) = \cos\omega t$ the long-time behavior ($t \gg 1/\Gamma$) of the above expression takes the following form:

$$\frac{W(t)}{W_0} = 1 + \begin{cases} -\sqrt{t/t_*}, & t \ll \tau_{so}, t_*, \\ (1/2)\sqrt{t/t_*}, & \tau_{so} \ll t \ll t_*, \end{cases} \qquad t_* = \frac{\pi^3\Gamma}{2\delta^2}. \quad (17)$$

These two limiting cases differ by the presence or absence of the triplet contribution and correspond to the orthogonal or symplectic symmetry classes. Thus the weak localization correction to the classical absorption rate in the symplectic case is *positive* and its magnitude is half that for the orthogonal case. Exactly the same holds for the weak localization correction to the conductivity of a quasi-1d disordered wire [10], so the relation (3) is extended also to the symplectic case.

The perturbative result (17) is valid only for times $t \ll t_*$. At $t \gg t_*$ the relation (3) suggests to use the known results for a quasi-1d wire [17], where, in spite of the positive sign of the first weak localization correction, all states are localized, with the localization length four times larger than in the orthogonal case. Localization implies $D(\omega) \to 0$ at $\omega \to 0$, for which equation (3) gives $W(t) \to 0$ at $t \to \infty$.

4 Quantum kicked rotor with spin

The exact time dependence of the correction (17) is derived rigorously for the disordered model (4). However, one can expect that the increase of the energy absorption rate in time is a general result valid for an ergodic dynamical system possessing symplectic symmetry. In order to support this statement we introduce a spin degree of freedom into the standard kicked rotor model [18]. Correspondingly, $V(\theta)$ in the Hamiltonian (1) acquires a 2×2 matrix structure:

$$V(\theta) = \cos\alpha\cos\beta\cos\theta + \frac{\sigma^x}{2}\cos\alpha\sin\beta\sin 2\theta + \sigma^z\sin\alpha\sin\theta,$$

where the parameters α and β allow to switch between different symmetry classes. Note that the presence of

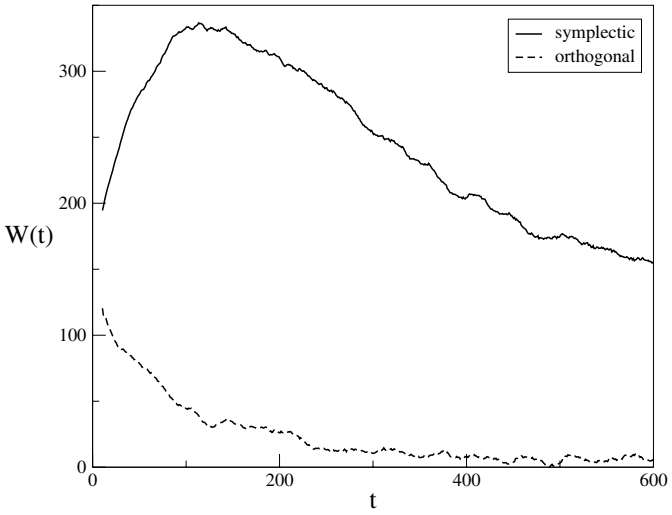

Fig. 2. The energy absorption rate as a function of time for the standard and symplectic kicked rotor. The energy $E(t)$ is averaged over 100 trajectories with different initial conditions.

both σ^x and σ^z terms with *different* dependencies on θ is essential, as otherwise the spin sector of the Hamiltonian could be diagonalized by a global spin rotation.

In order to observe the dynamical anti-localization we fix the parameters $\alpha = 0.187$, $\beta = 1.284$ which corresponds to the symplectic symmetry class [18] and study the evolution of the wave-packets in the momentum space. The other parameters are $T = 1$, $I = 10\,(\sqrt{5}-1)/(2\pi)$, $K = 10\,I$, and the Hilbert space size $\ell_{max} = 16384$. This model is very convenient for numerical study since the application of the Floquet operator to a state can be performed by using the Fast Fourier Transform algorithm as for the standard QKR model. Figure 2 shows that the energy absorption rate indeed initially increases in time until the strong localization changes this behavior to the opposite one. For comparison we plot the energy absorption rate for the standard QKR calculated for the same values of T, I and K as well. Here, in contrast, the quantum correction is negative from the very beginning.

5 Conclusions

The main results of the paper are represented by equation (17) and Figure 2. The former shows that the analogy between the energy absorption by an ac driven chaotic quantum dot and the propagation in a quasi-1d disordered wire is valid also in the presence of the spin-orbit interaction, i.e. for systems of the symplectic symmetry class, at least at the level of the first weak anti-localization correction. Numerical results for the quantum kicked rotor exhibit qualitatively the same behavior: weak dynamic anti-localization at shorter times, and strong dynamic localization at longer times. At the weak anti-localization stage the energy for both chaotic systems exhibits a peculiar super-Ohmic growth in time, in spite of the fact that no resonances are present in the considered systems.

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